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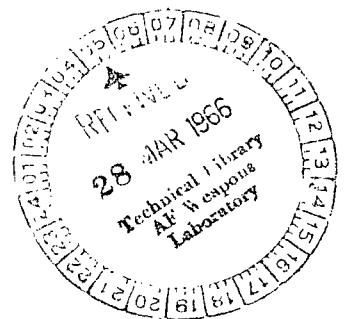
NASA TN D-3079

COMPARISON OF PHASE-COHERENT AND NON-PHASE-COHERENT CODED COMMUNICATIONS

by J. P. Strong III and T. V. Saliga

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1966

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ABSTRACT

The theoretical word-error probability versus signal-to-noise ratio has been evaluated for the following five communications systems:

1. Biorthogonal phase coherent, uncoded

2. Biorthogonal phase-coherent, word coded

3. Orthogonal phase-coherent, word coded

4. Orthogonal non-phase coherent, word coded

5. Same as (4) but with non-optimum detection.

Plots and tabulations of these results are presented. The performance of the coded systems are compared to each other and to the uncoded system. It is shown that systems (2), (3), and (4) are nearly equal in performance for large numbers of bits per codeword and all offer significant signal-to-noise gains over the uncoded system. If n is the number of information bits per word, then the precision of word synchronization relative to system (4) increases:

1. linearly with respect to n for the uncoded system, and

2. exponentially with respect to n for the phase-coherent coded systems.

CONTENTS

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INTRODUCTION

It is well known that encoding information into sets of sequences or code words and decoding them optimally can increase the probability of correct detection when the signal is perturbed by noise. One of the most effective known coding techniques to combat additive white gaussian noise makes use of orthogonal code words, that is, the cross-correlation between any two words is zero. A coded telemeter using pulsed tones (PFM) is presently being used on NASA spacecraft. This is an example of a non-phase coherent orthogonal system. Phase coherent coded PCM is being studied for possible use in future missions.

To judge the relative merits of coded systems each needs to be theoretically compared to a common parameter of importance to the designer. The ordinary performance measure is the word-error probability versus available signal-to-noise ratio. Curves have been separately published (References 1, 2, and 3) which show the probability of word-error versus signal-to-noise ratio for the following systems: biorthogonal phase-coherent, orthogonal phase-coherent, non-phase coherent orthogonal, and uncoded systems. However, although the curves showed the relative performance of a particular system under various degrees of encoding, the difference of the parameters used in presenting the curves made it very difficult to make a comparison between the coding systems. Therefore, it was found necessary to calculate the word-error probability curves for all of the above systems using a common parameter. This parameter is the ratio

$$\frac{\text{available signal energy per bit}}{\text{noise power per unit bandwidth}}.$$

Appendix A is a discussion about the use, interpretation, and measurement of this ratio. The word-error probability curves are presented in this report in graphical form and, in Appendix B, in tabular form. Curves showing a comparison of the coding systems relative to an uncoded reference system have been calculated and are presented. The uncoded reference system chosen for the comparisons was ordinary biorthogonal PCM.

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ASSUMPTIONS AND MATHEMATICAL MODELS

In the calculation of the probability of error curves, the following assumptions were made and apply to all of the coding systems considered.

First, the signal $V_{in}(t)$ received at the decoder was considered to be

$$V_{in}(t) = V_s(t) + V_n(t) , \qquad (1)$$

where $V_s(t)$ is a data signal of S watts and $V_n(t)$ is band-limited, white gaussian noise with a mean value of zero and a power (variance) of N watts.

The one-sided bandwidth of the noise is B cycles per second so that the power spectral density of the noise is N/B.

For PFM, the signal is a burst of $\sqrt{2S} \cos\left(w_i t + \theta\right)$ lasting for τ seconds. If the word has n bits of information, the time per bit is $T = \tau/n$, and the energy per bit becomes $E = ST = S\tau/n$. If a square wave is used as the signal, it would have a voltage magnitude of $\pm\sqrt{S}$.

Second, the assumption is made that ideal matched filters are used for each of the possible code words and a decision circuit selects the filter with the largest output at the end of the word burst as the one matched to the code word that was sent. Figure 1 shows this detection scheme. Figures 2 through 6 present the probability of word-error curves for (1) a phase-coherent biorthogonal system, (2) a phase-coherent orthogonal system, (3) a non-phase-coherent orthogonal system, (4) a non-phase coherent orthogonal system using square wave signals, and (5) an uncoded biorthogonal phase coherent system. The non-phase coherent orthogonal system using square wave signals uses simple but non-optimum correlation techniques for detection. This type of system because of its simplicity is of special interest to GSFC programs. The following is a discussion of each type of coding and detection system as well as the equations used in calculating the probability of error curves.

Phase-Coherent Systems

For phase-coherent PFM, coded PCM, or for any coding system where the signal can be generated on the ground in precisely the same time phase as the incoming signal, the correlator circuit of Figure 7 may be used as the matched filter. It should be noted that the code words may have any waveshape provided the phase coherence and orthogonality assumptions hold.
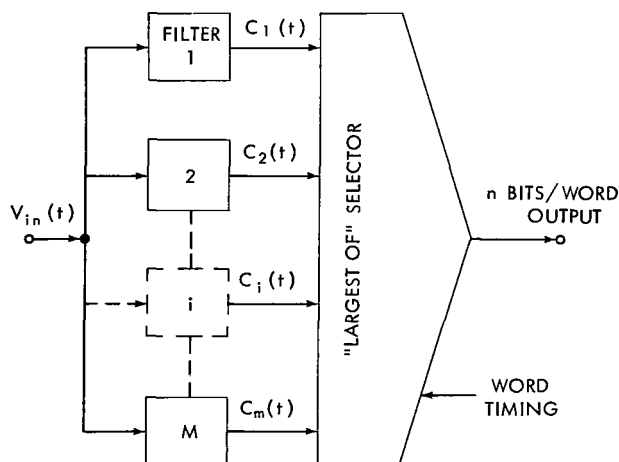

Figure 1—Optimum decoder for "M" equally likely codewords.
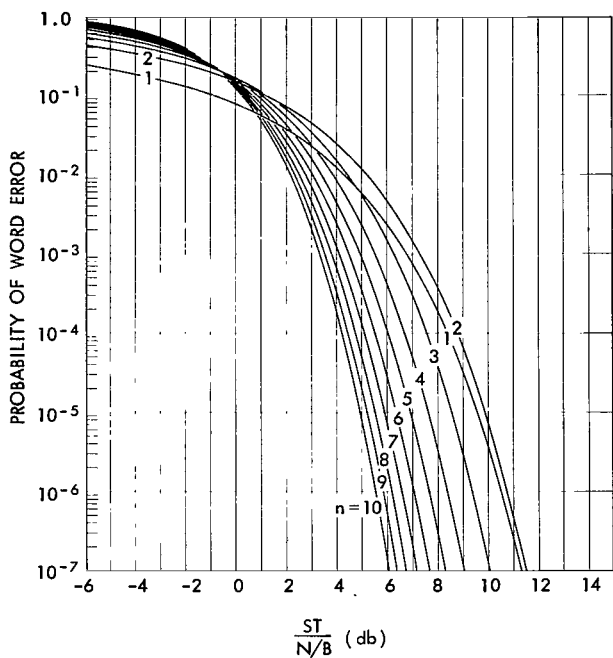
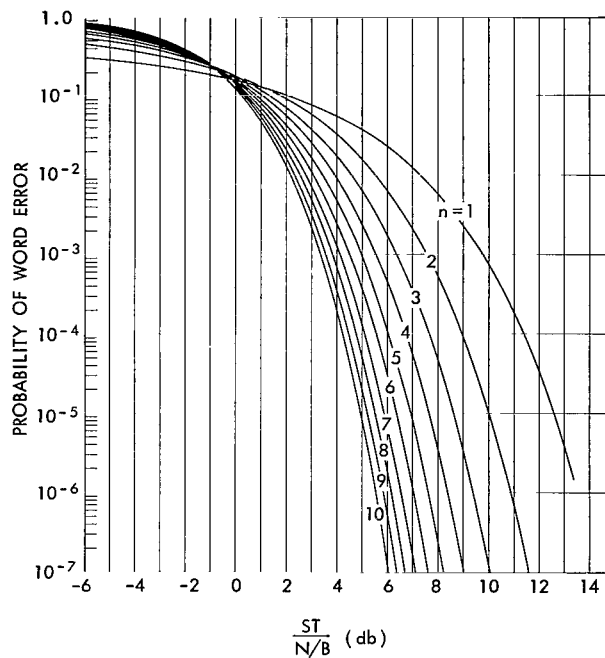Figure 2—Phase-coherent biorthogonal coding.
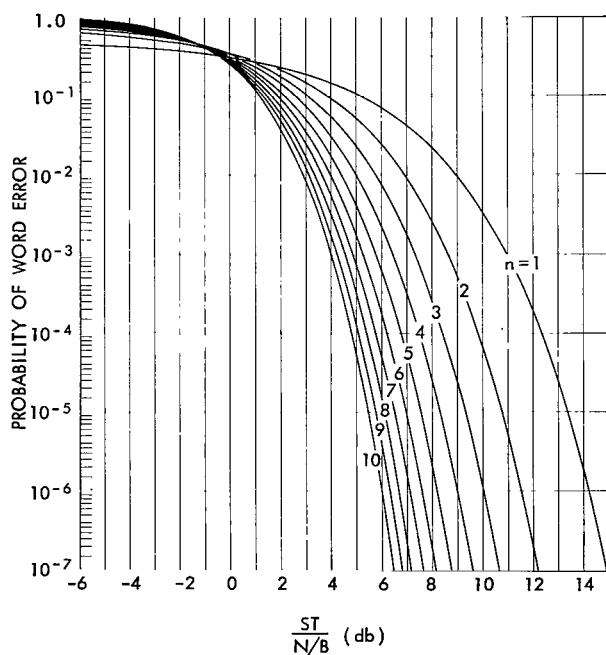

Figure 3—Phase-coherent orthogonal coding.

Figure 4—Non-phase-coherent orthogonal coding.
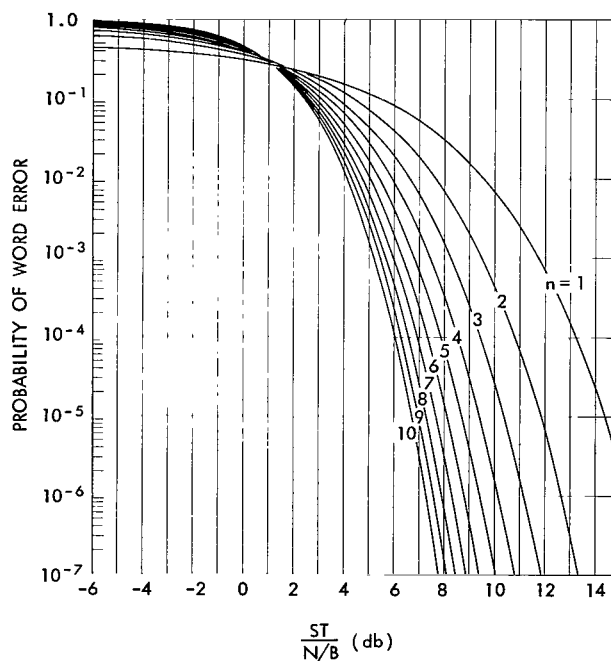
Figure 5—Non-phase-coherent orthogonal coding (non-optimal detection).

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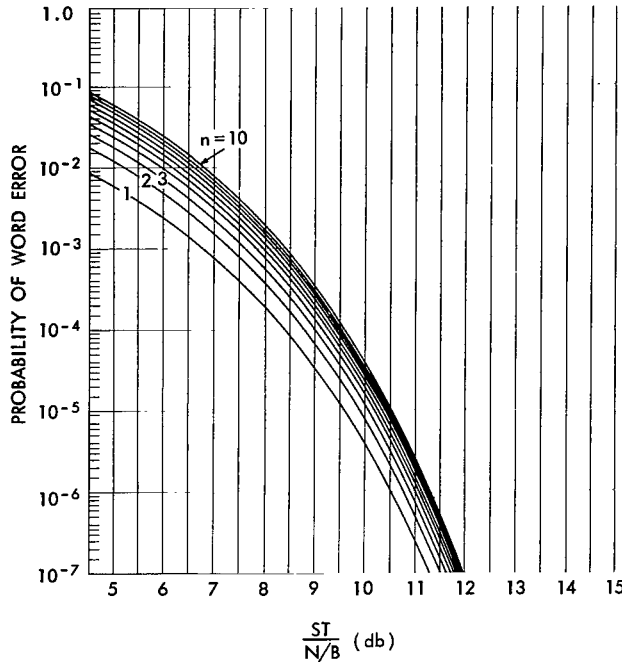
Figure 6—Uncoded phase-coherent biorthogonal.
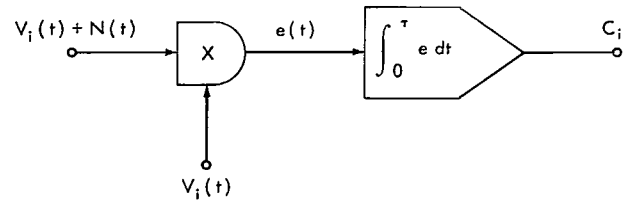
Figure 7—Matched filter for phase-coherent detection of the "i^{th}" codeword.

If $-V_i(t) + V_n(t)$ is placed at the input of the correlator of Figure 7 the output will be the negative of that obtained from $V_i(t) + V_n(t)$. This detector can detect complementary signals, and therefore only half as many correlators are needed for detection when complementary signals are sent. Thus an n-bit biorthogonal code requires only 2^{n-1} correlators.

The error probability equation for phase coherent biorthogonal systems is:

$$P_w(\epsilon) = 1 - \left\{ \frac{1}{\sqrt{\pi}} \int_{-0}^{\infty} e^{-(x-\sqrt{nV})^2} \left[\frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-y^2} dy \right]^{2^{n-1}-1} dx \right\} \qquad (2)$$

where:

$$V = \frac{ST}{N/B}$$

n = number of information bits per codeword.

Phase-coherent orthogonal codeword systems have a theoretical probability of word error given by:

$$P_w(\epsilon) = 1 - \left\{ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-(x-\sqrt{nV})^2} \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-y^2} dy \right]^{2^n-1} dx \right\} \qquad (3)$$

where:

$$V = \frac{ST}{N/B}$$

n = number of information bits per codeword.

Non-Phase-Coherent Orthogonal Systems

For non-phase-coherent orthogonal systems using sinusoidal bursts for codewords, the matched filter consists of the inphase and quadrature-phase correlators as shown in Figure 8 (Reference 4).

The signal phase angle, θ, is assumed to be of random value from codeword to codeword, but is constant over the period of any one codeword. The output of the non-coherent matched filter (c_i) is independent of θ.

The probability of word-error equation for non-phase-coherent orthogonal codewords (PFM) is:

$$P_w(\epsilon) = 1 - \int_0^\infty \left\{ \left[\int_0^x 2ye^{-y^2} dy \right]^{2^n-1} \frac{1}{2\pi} \int_0^{2\pi} 2xe^{-(x^2+nV-2\sqrt{nV}x\cos\theta)} d\theta \right\} dx , \qquad (4)$$

where again

$$V = \frac{ST}{N/B}$$

n = number of information bits per codeword.

If the signal were a square wave rather than a sinusoid, then, the optimum non-phase coherent detector is more complex. A pair of multipliers and integrators is required for the fundamental and each harmonic. The resultant output is the sum of the squares of each integrator output. In the interest of simpler square wave decoding, a non-optimum detection scheme has also been studied. This detection system is the same as that in Figure 8 except for the multiplying signals and the method of combining the two integrator outputs. The local signals are square waves of the appropriate frequency with a one-quarter-cycle relative time shift. The resultant output (c_i) is the sum of the absolute values of the integrator outputs.

$$C_i = |x_i| + |y_i|$$

A study of the autocorrelation function of a square wave and this combination·method will reveal that the output C_i is independent of the time shift between the input square wave and the local multiplying signals. The probability of error versus $ST/(N/B)$ has also been evaluated for decoders using this type of detection.
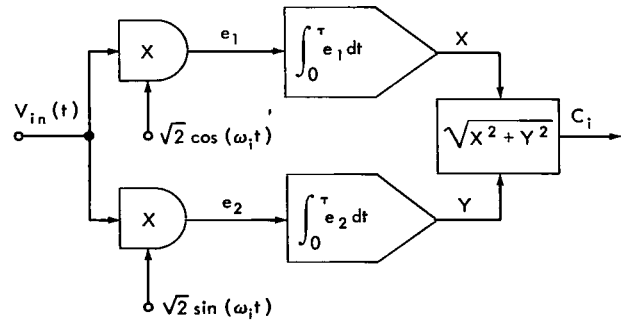
where: $V_{in}(t) = \sqrt{2S} \cos(\omega_i t + \theta) + V_n(t)$ for $0 \le t \le \tau$

Figure 8—Matched filter for non-phase-coherent detection of the "i^{th}" PFM codeword.

The word error probability equation for a non-phase-coherent orthogonal square-wave tone-burst system using these non-optimum correlators in the detector is:

$$P_w(\epsilon) = 1 - \int_0^\infty \left\{ \left[1 - \int_{-\pi/4}^{\pi/4} \frac{4}{\pi} e^{-x^2/2 \sin^2\theta} \, d\theta \right]^{2^n-1} \left[e^{-A^2/2} + e^{-B^2/2} \right] \cdot \frac{1}{\pi} \left[\int_{-A}^{B} e^{-y^2/2} \, dy \right] \right\} dx \, , \qquad (5)$$

where

$$A = x + \sqrt{nV}$$

$$B = x - \sqrt{nV}$$

n = number of information bits per codeword

V = ST/(N/B)

Uncoded System

A biorthogonal phase-coherent system which transmits one bit per code word and detects on a bit by bit basis is defined as being an uncoded system. This is ordinary PCM and for the non-return zero (NRZ) case, the matched filter is as shown in Figure 9. If the probability of the correct detection of one bit is $P_B(c)$, then the probability that n consecutive bits are correct is $[P_B(c)]^n$ and the word error probability for n-bit words is $1 - [P_B(c)]^n$ or $1 - [1 - P_B(e)]^n$ where $P_B(e)$ is the bit error probability for a one-bit biorthogonal coding system. Thus, we have
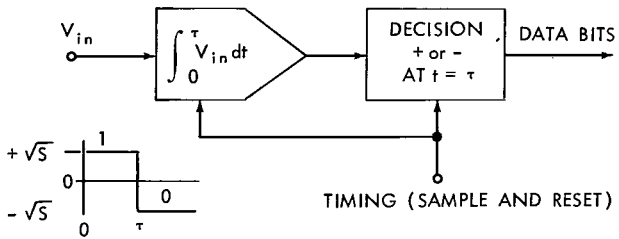
Figure 9—Matched filter and decision for NRZ PCM.

$$P_w(\epsilon) = 1 - \left[\frac{1}{\sqrt{\pi}} \int_0^\infty e^{-(x-\sqrt{v})^2} \, dx \right]^n \qquad (6)$$

as the probability of word error for an uncoded n-bit system.

COMPARISON OF CODED SYSTEMS

Figures 10 through 19 are plots comparing the ideal performance of each of the coded systems with the uncoded system.

The relative performance of each system is a function of word error probability. For convenience, the ST/(N/B) improvement is also interpreted as an increase in communication distance
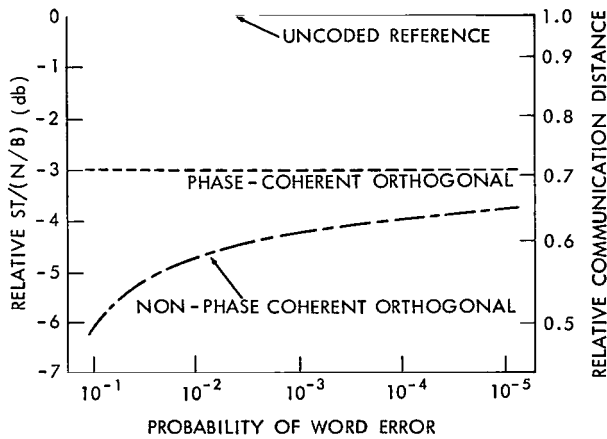
Figure 10—Comparison of n = 1 systems.

Figure 11 —Comparison of n = 2 systems. (Note: 2-Bit Biorthogonal Coding is Equivalent to a 2-Bit Uncoded Message.)

Figure 12—Comparison of n = 3 systems.
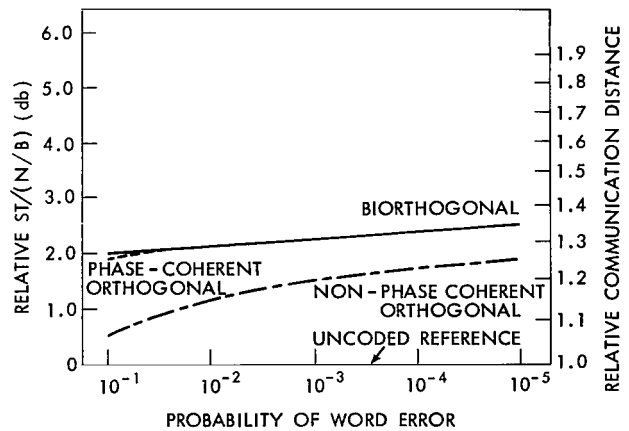
Figure 13—Comparison of n = 4 systems.

Figure 14—Comparison of n = 5 systems.
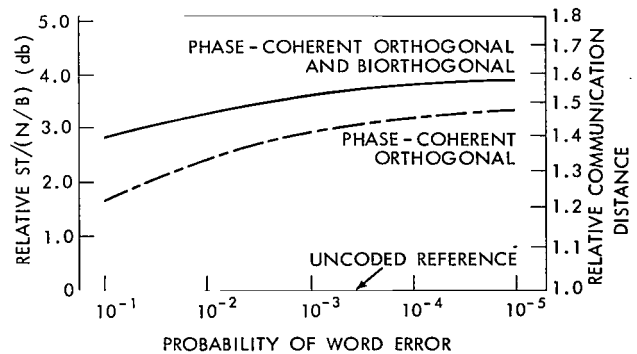
Figure 15—Comparison of n = 6 systems.

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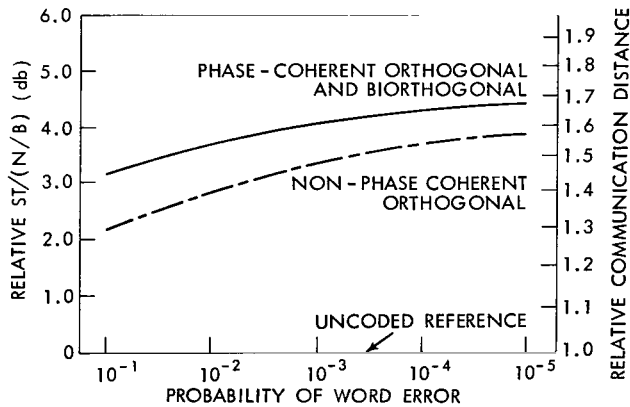
Figure 16—Comparison of n = 7 systems.
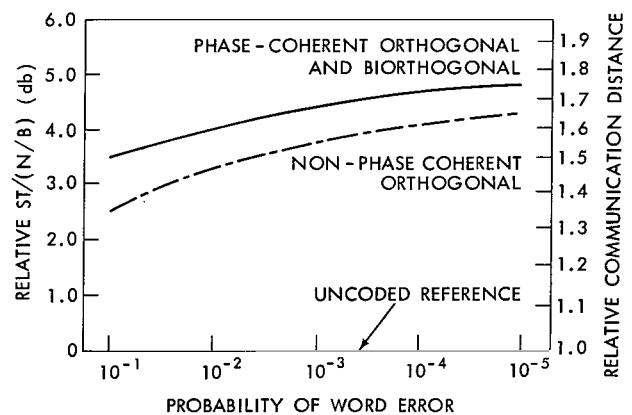
Figure 17—Comparison of n = 8 systems.
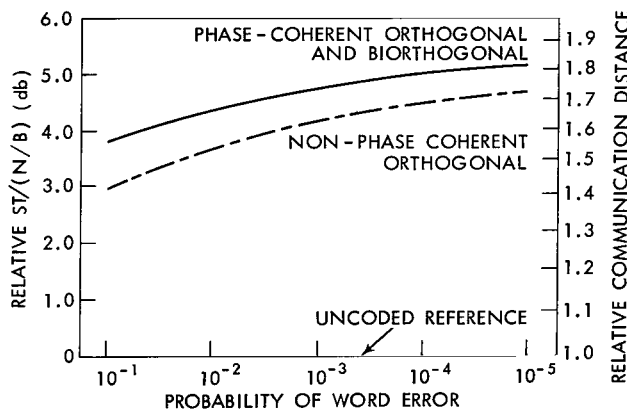
Figure 18—Comparison of n = 9 systems.
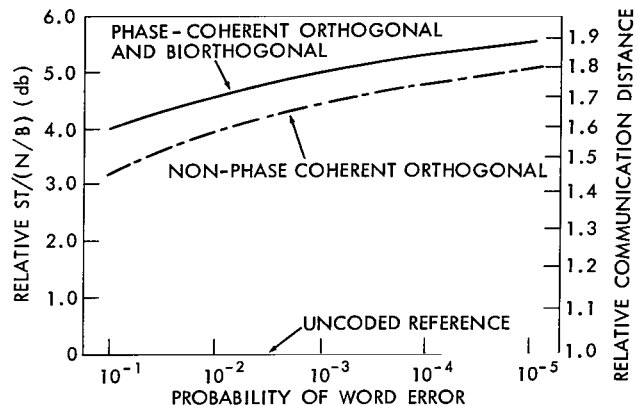
Figure 19—Comparison of n = 10 systems.

relative to that of an ideal uncoded system. The square wave PFM system has not been plotted in these curves since it is a non-ideal, special case.

Several general observations may be made concerning these graphs. First, for n = 2 and 3 there is little or no improvement by use of a coded system, whereas for higher values of n, the improvements begin to be worthwhile. Secondly, for n = 4 or more, the performance of orthogonal and bi-orthogonal coherent systems are virtually identical. Thus, the choice of either of these two coherent systems should be solely dependent on bandwidth and ease of implementation considerations, for which the biorthogonal system is generally favored. Furthermore, it may be noted that the non-phase coherent system is only about 1db or less poorer than the coherent system. For larger n, it is only about 0.6db poorer. To the extent that 0.6db is small to the user, the following statement can be made. For n more than 6, the coding gains achieved by use of phase-coherent-orthogonal, biorthogonal or non-phase coherent-orthogonal, encoded words are closely equal for any given n. Thus the choice of any of these coding schemes for use in a communications system is dependent only on their relative ease of synchronization and implementation, and their bandwidth requirements.

SYNCHRONIZATION CONSIDERATIONS

As the art of telemetry has progressed, it has become more evident that synchronization is not an engineering detail, but is a fundamental communications problem. The codings evaluated in this report all assume perfect timing. Yet poor synchronization will nullify any coding improvement.

There is an extensive range of synchronization problems. There are the problems of format sync, frame sync, word sync, symbol or bit sync, and finally R-F phase sync. Orthogonal word encoding demands a much larger number of symbols per word than an uncoded system. Thus, the problems of word and symbol sync are aggravated. Non-phase-coherent PFM is a notable exception to this rule. Since envelope detection is employed, symbol sync is not required. As a matter of fact, an uncoded serial NRZ-PCM system needs to sync an n-bit word n times more accurately than the non-coherent system for a given $ST/(N/B)$ degradation. The coded phase-coherent systems generally require the more efficient, precise word and symbol synchronization. The literature is abundant with sync studies and techniques. No attempt will be made to describe them here, but the relative difficulty of word and symbol synchronization for these systems will be pointed out.

When a timing error exists between the decoder signals and the data words, two things occur which increase the probability of error. First, the signal correlator output voltage is reduced from its maximum, making the input $ST/(N/B)$ appear worse than it actually is. Secondly, timing errors will usually destroy true word orthogonality by allowing small correlations to build up in non-signal correlators. For phase-coherent systems, the reduction in desired signal correlator voltage versus timing error is just the auto-correlation function of the serial data word stream. For non-phase-coherent systems, this reduction is the auto-correlation function of the word envelope. The relative word sync accuracies required for each of the systems previously analyzed will be illustrated for specific codes by comparing their respective data auto-correlation functions.

Let the uncoded system be a serial binary NRZ-PCM system composed of n bits per word. The phase-coherent orthogonal and biorthogonal systems may be constructed using codewords composed of binary symbols. An orthogonal codeword dictionary must contain 2^n words and each word can be constructed with 2^n binary symbols (as a lower bound). The biorthogonal dictionary requires only 2^{n-1} of these codewords and their complements. The codewords for the non-phase-coherent orthogonal systems have already been specified.

These particular systems all have the same triangular-shaped data auto-correlation function, but when transmitting at the same information bit-rate, their time sharpness is quite different. Figure 20 illustrates these differences. Therefore, for a specified minimum C/C_{max}:

1. The uncoded system must have word sync n times better than a non-phase-coherent orthogonal system such as PFM.

2. A phase-coherent biorthogonal system must have word sync 2^{n-1} times better than the non-phase-coherent orthogonal system.

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3. A phase-coherent orthogonal system must have word sync 2^n times better than the non-phase-coherent orthogonal system.

If the PCM symbols are split-phase coded, then the auto-correlation functions are further sharpened by a factor of three.

CONCLUSIONS

It is hoped that the results and comparisons of the communications systems treated in this report will aid the telemetry designer in his choice of a communications system. The results show that both phase-coherent and non-phase-coherent word coded systems give significant signal-to-noise improvements in digital space communications. Also, for about six or more bits per codeword, the ideal systems are nearly identical in performance. Furthermore, biorthogonal systems are more hardware and bandwidth conservative than the orthogonal coherent systems. These factors lead to the following conclusion: the choice of an orthogonal, non-phase coherent (PFM) or a biorthogonal system depends primarily on which is less costly in bandwidth and hardware, including synchronization requirements.

A complete biorthogonal coded telemeter with efficient synchronization has not yet been demonstrated to the authors' knowledge. It is therefore difficult to judge which of the above systems will ultimately be less costly to the user. However, with the use of efficient cyclic code generators, biorthogonal coded communications could become competitive with PFM systems, particularly for large numbers of bits per codeword.
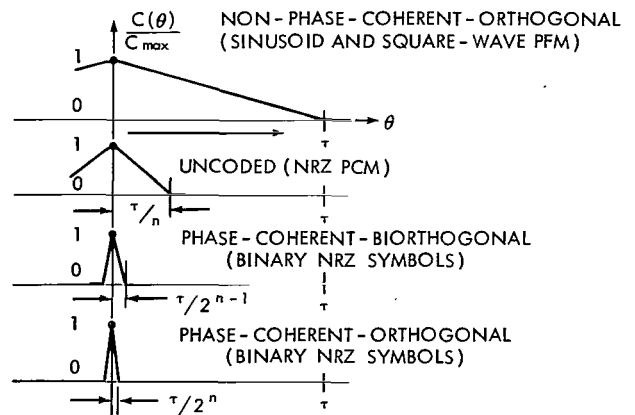
where: θ = TIMING ERROR IN SEC.
τ = WORD PERIOD IN SEC.
n = NUMBER OF INFORMATION BITS PER WORD
$\frac{C(\theta)}{C_{max}}$ = SIGNAL CORRELATOR OUTPUT NORMALIZED TO ITS MAXIMUM

Figure 20—Data auto-correlation functions for five communication systems.

ACKNOWLEDGMENTS

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Appendix A

The Use, Interpretation, and Measurement of the Parameter ST/(N/B)

To provide a meaningful comparison of the various communication systems, a common reference parameter must be chosen which is indicative of the cost to the system user. This parameter should be easily measured for any given system.

The parameters ordinarily of importance in the digital space communications problem are the cost per bit of information transmitted, with specified bit or word-error probabilities and noise conditions.

Since additive white gaussian noise is the primary perturbation in the space channel, and linear synchronous radio frequency detection is usually employed at GSFC tracking stations, then the noise power per unit bandwidth at the input to the data decoder is the important noise parameter. These conditions justified the use of the same parameter that Sanders used in his comparison of communication systems (Reference 1), and subsequently employed by Rochelle (Reference 2) and Viterbi (Reference 3), namely:

$$\frac{ST}{N/B} = \frac{\text{Signal energy per bit}}{\text{noise power per unit bandwidth}} \text{ at the decoder input },$$

where

S = signal power in watts

T = period of one bit in seconds assuming serial data

N = noise power in watts

B = single-sided equivalent noise bandwidth

An interpretation more directly related to word encoded systems is

$$\frac{ST}{N/B} = \frac{\text{available energy per word/number of data bits per word}}{\text{noise power per unit bandwidth}}.$$

Another useful interpretation is

$$\frac{ST}{N/B} = \text{signal/noise power ratio in a bandwidth equal to the bit rate }.$$

This may be shown as follows:

$$\frac{ST}{N/B} = \frac{S}{N}\frac{B}{1/T}, \text{ but } 1/T = \text{bit rate and let } B = 1/T \text{ (cps) .}$$

Before an attempt is made to measure $ST/(N/B)$ in a communications system, it is important to check that the system conforms to the assumptions made for the mathematical model. These are:

1. each bit or code word has equal energy.

2. The noise has constant density over all frequencies or at least over all frequencies to which the matched filters in the decoder have a significant response.

3. The noise has gaussian statistics.

The measurement technique of $ST/(N/B)$ will be demonstrated by way of an example uncoded PCM system. A simple extension of this method allows measurement of $ST/(N/B)$ for any word encoded system which meets the above conditions.
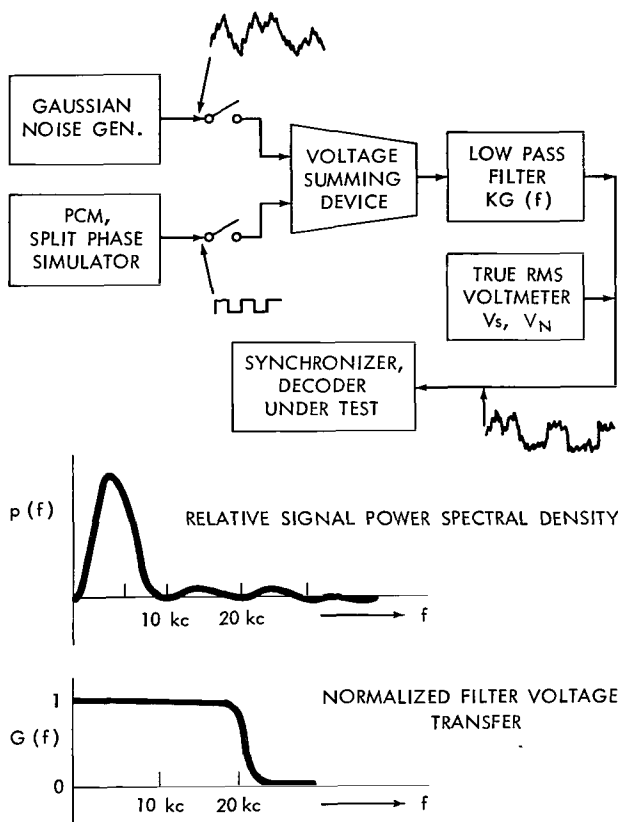
Figure A-1—Measurement of ST/(N/B).

Suppose the bit-error probability of a split-phase, uncoded PCM system was to be found experimentally for a particular decoding system. To compare signal to noise performance with that of the theoretical optimum, it is necessary to calculate $ST/(N/B)$ for the experimental system parameters used.

Figure A-1 is a block diagram of the set up. The bit rate will be chosen as 5000 bits per second. The noise generator is presumed to have a constant noise density over the passband of the filter.

For a given signal and noise level setting, there will be a corresponding signal voltage (V_s) and noise voltage (V_n) read on the true rms voltmeter (when switched in separately).

The low-pass filter is assumed to have a flat response out to near its -3db frequency (f_0); 20 kilocycles in this example. The equivalent noise bandwidth of this filter must be found to determine the noise power per unit bandwidth.

If the filter has a gain K over its flat region, and a frequency transfer function $KG(f)$, then

$$B = \frac{1}{K^2} \int_0^\infty \left[KG(f) \right]^2 df = \int_0^\infty G^2(f) df$$

For a simple R-C filter, $B = \pi/2 \, f_0$ (cps). For multi-section, maximally flat filters, B will usually be somewhere between f_0 and $\pi/2 \, f_0$. With these parameters known, $ST/(N/B)$ may be found

$$\frac{ST}{N/B} = \frac{S}{N} TB = \left(\frac{V_s}{V_n} \right)^2 \times \frac{1}{(\text{bit rate})} \times B$$

$$= \left(\frac{V_s}{V_n} \right)^2 \frac{B}{5000}$$

If B were measured as 20 kilocycles, V_s as 1.50 volts rms and V_n as 1.00 volt, then $ST/(N/B) = 9.0$ and $ST/(N/B)$ in db $= 10 \log 9 = 9.54$db.

It should be noted that the low-pass filter used in this example cuts off some harmonic signal energy. For this case, deleting signal energy above four times the bit rate excludes about 7 percent of the available energy. This causes at most a 0.3db measurement error if the intended system has a larger bandwidth.

$ST/(N/B)$ measurement for word coded systems is essentially the same. The ST product is found by measuring the total code word energy available at the decoder input and dividing this by the number of information bits which the code word represents. The noise density N/B may be found using the same methods as the example.

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A-2. Rochelle, R. W., "Pulse Frequency Modulation Telemetry," NASA Technical Report R-189, January 1964.

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Appendix B

Probability of Word Error Tables

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Table B-1

Phase-Coherent Biorthogonal Systems.

ST/(N/B)* (db)	n = 1**	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
-6	2.39×10^{-1}	4.21×10^{-1}	5.29×10^{-1}	6.03×10^{-1}	6.60×10^{-1}	7.06×10^{-1}	7.45×10^{-1}	7.77×10^{-1}	8.05×10^{-1}	8.29×10^{-1}
-5	2.13	3.81	4.77	5.43	5.94	6.36	6.72	7.03	7.30	7.55
-4	1.86	3.38	4.20	4.75	5.17	5.52	5.82	6.09	6.33	6.55
-3	1.58	2.92	3.58	3.99	4.30	4.55	4.76	4.95	5.13	5.29
-2	1.31	2.44	2.93	3.19	3.37	3.49	3.60	3.69	3.78	3.85
-1	1.04	1.97	2.28	2.39	2.43	2.45	2.45	2.44	2.43	2.42
0	7.87×10^{-2}	1.51	1.66	1.64	1.59	1.51	1.44	1.37	1.30	1.24
1	5.63	1.09	1.11	1.01	9.01×10^{-2}	7.94×10^{-2}	6.99×10^{-2}	6.16×10^{-2}	5.44×10^{-2}	4.82×10^{-2}
2	3.75	7.36×10^{-2}	6.67×10^{-2}	5.42×10^{-2}	4.28	3.36	2.63	2.07	1.64	1.30
3	2.29	4.52	3.51	2.42	1.62	1.08	7.17×10^{-3}	4.78×10^{-3}	3.21×10^{-3}	2.16×10^{-3}
4	1.25	2.48	1.56	8.61×10^{-3}	4.59×10^{-3}	2.43×10^{-3}	1.29	6.82×10^{-4}	3.64×10^{-4}	1.95×10^{-4}
5	5.95×10^{-3}	1.19	5.61×10^{-3}	2.30	9.04×10^{-4}	3.52×10^{-4}	1.37×10^{-4}	5.34×10^{-5}	2.09×10^{-5}	8.20×10^{-6}
6	2.39	4.77×10^{-3}	1.54	4.27×10^{-4}	1.13	2.95×10^{-5}	7.66×10^{-6}	1.99×10^{-6}	5.19×10^{-7}	1.36×10^{-7}
7	7.73×10^{-4}	1.54	3.06×10^{-4}	5.09×10^{-5}	8.07×10^{-6}	1.26×10^{-6}	1.95×10^{-7}	3.02×10^{-8}		
8	1.91	3.82×10^{-4}	4.00×10^{-5}	3.49×10^{-6}	2.89×10^{-7}	2.34×10^{-8}				
9	3.36×10^{-5}	6.73×10^{-5}	3.14×10^{-6}	1.21×10^{-7}						
10	3.87×10^{-6}	7.75×10^{-6}	1.29×10^{-7}							
11	2.61×10^{-7}	5.23×10^{-7}								

*ST/(N/B) (db) ⊟ $10 \log_{10}$ (signal energy per bit/noise power per unit bandwidth).

**n = The number of information bits per codeword.

Table B-2

Phase-Coherent Orthogonal Systems.

ST/(N/B)* (db)	n = 1**	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
-6	3.08×10^{-1}	4.47×10^{-1}	5.39×10^{-1}	6.07×10^{-1}	6.62×10^{-1}	7.07×10^{-1}	7.45×10^{-1}	7.77×10^{-1}	8.05×10^{-1}	8.29×10^{-1}
-5	2.87	4.09	4.88	5.48	5.96	6.36	6.72	7.03	7.31	7.55
-4	2.64	3.67	4.31	4.80	5.19	5.52	5.82	6.09	6.33	6.55
-3	2.39	3.22	3.70	4.04	4.32	4.56	4.76	4.95	5.13	5.29
-2	2.14	2.74	3.05	3.24	3.39	3.51	3.61	3.70	3.78	3.85
-1	1.86	2.25	2.39	2.44	2.46	2.46	2.45	2.44	2.43	2.42
0	1.59	1.77	1.75	1.68	1.60	1.52	1.44	1.37	1.30	1.24
1	1.31	1.32	1.19	1.04	9.13×10^{-2}	7.99×10^{-2}	7.01×10^{-2}	6.17×10^{-2}	5.45×10^{-2}	4.82×10^{-2}
2	1.04	9.12×10^{-2}	7.23×10^{-2}	5.62×10^{-2}	4.35	3.38	2.64	2.08	1.64	1.30
3	7.89×10^{-2}	5.78	3.85	2.53	1.65	1.09	7.20×10^{-3}	4.79×10^{-3}	3.21×10^{-3}	2.16×10^{-3}
4	5.65	3.27	1.74	9.04×10^{-3}	4.70×10^{-3}	2.46×10^{-3}	1.29	6.84×10^{-4}	3.64×10^{-4}	1.95×10^{-4}
5	3.77	1.61	6.32×10^{-3}	2.42	9.28×10^{-4}	3.57×10^{-4}	1.38×10^{-4}	5.35×10^{-5}	2.09×10^{-5}	8.20×10^{-6}
6	2.30	6.66×10^{-3}	1.76	4.54×10^{-4}	1.16	2.99×10^{-5}	7.71×10^{-6}	2.00×10^{-6}	5.20×10^{-7}	
7	1.26	2.21	3.52×10^{-4}	5.43×10^{-5}	8.33×10^{-6}	1.28×10^{-6}				
8	6.00×10^{-3}	5.57×10^{-4}	4.64×10^{-5}	3.74×10^{-6}						
9	2.41	9.93×10^{-5}	3.65×10^{-6}							
10	7.83×10^{-4}	1.15								
11	1.94	7.82×10^{-7}								
12	3.43×10^{-5}									
13	3.97×10^{-6}									

*ST/(N/B) (db) $=$ 10 $\log_{10}$ (signal energy per bit/noise power per unit bandwidth).

**n $=$ The number of information bits per codeword.

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Table B-3

Non-Phase-Coherent Orthogonal Systems.

ST/(N/B)* (db)	n = 1**	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
-6	4.41×10^{-1}	6.23×10^{-1}	7.15×10^{-1}	7.73×10^{-1}	8.13×10^{-1}	8.43×10^{-1}	8.67×10^{-1}	8.87×10^{-1}	9.03×10^{-1}	9.16×10^{-1}
-5	4.27	5.93	6.76	7.28	7.65	7.94	8.18	8.38	8.55	8.70
-4	4.10	5.57	6.28	6.72	7.04	7.29	7.51	7.69	7.86	8.00
-3	3.89	5.14	5.70	6.03	6.27	6.46	6.61	6.75	6.88	6.99
-2	3.65	4.64	5.02	5.21	5.33	5.42	5.49	5.54	5.59	5.64
-1	3.36	4.07	4.25	4.28	4.26	4.22	4.18	4.13	4.08	4.03
0	3.03	3.44	3.41	3.28	3.12	2.96	2.82	2.68	2.54	2.42
1	2.66	2.77	2.55	2.28	2.03	1.80	1.60	1.43	1.27	1.14
2	2.26	2.10	1.74	1.40	1.12	9.01×10^{-2}	7.23×10^{-2}	5.82×10^{-2}	4.69×10^{-2}	3.79×10^{-2}
3	1.84	1.47	1.05	7.29×10^{-2}	5.03×10^{-2}	3.47	2.40	1.66	1.15	7.98×10^{-3}
4	1.42	9.23×10^{-2}	5.38×10^{-2}	3.05	1.71	9.52×10^{-3}	5.30×10^{-3}	2.96×10^{-3}	1.65×10^{-3}	9.21×10^{-4}
5	1.03	5.09	2.25	9.61×10^{-3}	4.03×10^{-3}	1.68	6.96×10^{-4}	2.88×10^{-4}	1.19×10^{-4}	4.93×10^{-5}
6	6.83×10^{-2}	2.37	7.25×10^{-3}	2.12	6.03×10^{-4}	1.70×10^{-4}	4.74×10^{-5}	1.32×10^{-5}	3.65×10^{-6}	1.01×10^{-6}
7	4.08	8.87×10^{-3}	1.67	2.96×10^{-4}	5.08×10^{-5}	8.56×10^{-6}	1.43×10^{-6}	2.37×10^{-7}		
8	2.13	2.53	2.53×10^{-4}	2.35×10^{-5}	2.10×10^{-6}					
9	9.42×10^{-3}	5.09×10^{-4}	2.27×10^{-5}	9.27×10^{-7}						
10	3.37	6.66×10^{-5}	1.06×10^{-6}							
11	9.23×10^{-4}	5.06×10^{-6}								
12	1.81									
13	2.32×10^{-5}									
14	1.76×10^{-6}									

*ST/(N/B) (db) $= 10 \log_{10}$ (signal energy per bit/noise power per unit bandwidth).

**n = The number of information bits per codeword.

Table B-4

Non-Phase-Coherent Orthogonal, Square
Wave Codewords, Non-Optimally Detected Systems.

ST/(N/B)* (db)	n = 1**	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
-6	4.42×10^{-1}	6.28×10^{-1}	7.26×10^{-1}	7.90×10^{-1}	8.36×10^{-1}	8.71×10^{-1}	8.98×10^{-1}	9.20×10^{-1}	9.37×10^{-1}	9.50×10^{-1}
-5	4.29	6.00	6.91	7.51	7.97	8.34	8.64	8.88	9.08	9.25
-4	4.12	5.66	6.47	7.03	7.48	7.85	8.16	8.43	8.66	8.86
-3	3.92	5.26	5.95	6.44	6.85	7.20	7.51	7.79	8.03	8.25
-2	3.69	4.79	5.34	5.74	6.07	6.38	6.65	6.91	7.15	7.37
-1	3.41	4.26	4.65	4.92	5.15	5.36	5.57	5.76	5.95	6.14
0	3.10	3.67	3.87	3.99	4.10	4.19	4.29	4.39	4.49	4.59
1	2.74	3.04	3.06	3.02	2.99	2.96	2.94	2.93	2.93	2.93
2	2.36	2.40	2.25	2.08	1.94	1.82	1.71	1.62	1.55	1.48
3	1.96	1.77	1.50	1.27	1.08	9.24×10^{-2}	7.98×10^{-2}	6.96×10^{-2}	6.10×10^{-2}	5.38×10^{-2}
4	1.55	1.21	8.92×10^{-2}	6.57×10^{-2}	4.87×10^{-2}	3.65	2.76	2.11	1.63	1.26
5	1.16	7.46×10^{-2}	4.54	2.76	1.69	1.04	6.51×10^{-3}	4.10×10^{-3}	2.61×10^{-3}	1.67×10^{-3}
6	8.15×10^{-2}	4.04	1.90	8.90×10^{-3}	4.19×10^{-3}	1.99×10^{-3}	9.51×10^{-4}	4.59×10^{-4}	2.24×10^{-4}	1.10×10^{-4}
7	5.26	1.87	6.25×10^{-3}	2.07	6.90×10^{-4}	2.31×10^{-4}	7.80×10^{-5}	2.66×10^{-5}	9.12×10^{-6}	3.15×10^{-6}
8	3.07	7.06×10^{-3}	1.52	3.24×10^{-4}	6.91×10^{-5}	1.49×10^{-5}	3.22×10^{-6}	7.06×10^{-7}	1.56×10^{-7}	3.48×10^{-8}
9	1.58	2.09	2.57×10^{-4}	3.13×10^{-5}	3.82×10^{-6}	4.71×10^{-7}	5.88×10^{-8}			
10	6.96×10^{-3}	4.55×10^{-4}	2.78×10^{-5}	1.68×10^{-6}	1.02×10^{-7}					
11	2.54	6.85×10^{-5}	1.73×10^{-6}	4.36×10^{-8}						
12	7.31×10^{-4}	6.50×10^{-6}	5.46×10^{-8}							
13	1.57	3.48×10^{-7}								
14	2.35×10^{-5}									
15	2.23×10^{-6}									

*ST/(N/B) (db) $= 10 \log_{10}$ (signal energy per bit/noise power per unit bandwidth).

**n $=$ The number of information bits per codeword.

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Table B-5

Uncoded, Biorthogonal Systems.

ST/(N/B)* (db)	n = 1**	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
-6	2.39×10^{-1}	4.21×10^{-1}	5.60×10^{-1}	6.65×10^{-1}	7.45×10^{-1}	8.06×10^{-1}	8.53×10^{-1}	8.88×10^{-1}	9.15×10^{-1}	9.35×10^{-1}
-5	2.13	3.81	5.13	6.17	6.99	7.63	8.13	8.53	8.85	9.09
-4	1.86	3.38	4.61	5.61	6.43	7.09	7.63	8.08	8.43	8.73
-3	1.58	2.92	4.04	4.98	5.78	6.45	7.01	7.48	7.89	8.22
-2	1.31	2.44	3.43	4.29	5.03	5.68	6.25	6.74	7.16	7.53
-1	1.04	1.97	2.80	3.55	4.22	4.82	5.36	5.84	6.27	6.66
0	7.87×10^{-2}	1.51	2.18	2.79	3.36	3.88	4.36	4.81	5.22	5.59
1	5.63	1.09	1.60	2.07	2.52	2.94	3.33	3.71	4.06	4.40
2	3.75	7.36×10^{-2}	1.08	1.42	1.74	2.05	2.35	2.64	2.91	3.18
3	2.29	4.52	6.71×10^{-2}	8.84×10^{-2}	1.09	1.30	1.50	1.69	1.88	2.07
4	1.25	2.49	3.70	4.91	6.10×10^{-2}	7.27×10^{-2}	8.43×10^{-2}	9.57×10^{-2}	1.07	1.18
5	5.95×10^{-3}	1.19	1.78	2.36	2.94	3.52	4.09	4.67	5.23×10^{-2}	5.80×10^{-2}
6	2.39	4.77	7.15×10^{-3}	9.52×10^{-3}	1.19	1.42	1.66	1.90	2.13	2.36
7	7.73×10^{-4}	1.55×10^{-3}	2.32	3.09	3.86×10^{-3}	4.63×10^{-3}	5.40×10^{-3}	6.17×10^{-3}	6.94×10^{-3}	7.70×10^{-3}
8	1.91	3.82×10^{-4}	5.73×10^{-4}	7.64×10^{-4}	9.54×10^{-4}	1.14	1.34	1.53	1.72	1.91
9	3.36×10^{-5}	6.73×10^{-5}	1.01	1.35	1.68	2.02×10^{-4}	2.36×10^{-4}	2.70×10^{-4}	3.03×10^{-4}	3.36×10^{-4}
10	3.87×10^{-6}	7.75×10^{-6}	1.16×10^{-5}	1.55×10^{-5}	1.94×10^{-5}	2.32×10^{-5}	2.71×10^{-5}	3.10×10^{-5}	3.49×10^{-5}	3.87×10^{-5}
11	2.61×10^{-7}	5.23×10^{-7}	7.84×10^{-7}	1.05×10^{-6}	1.31×10^{-6}	1.57×10^{-6}	1.83×10^{-6}	2.09×10^{-6}	2.35×10^{-6}	2.61×10^{-6}
12				3.60×10^{-8}	4.50×10^{-8}	5.40×10^{-8}	6.30×10^{-8}	7.21×10^{-8}	8.11×10^{-8}	9.01×10^{-8}

*ST/(N/B) (db) $= 10 \log_{10}$ (signal energy per bit/noise power per unit bandwidth).

**n = The number of information bits per codeword.

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